Coupling between two resonant waves in a waveguide free-electron laser

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(Received 5 August 1996)

The coupling between two resonant branches of a waveguide free-electron laser is studied. It is shown that the induced electron energy spread, which is produced along with the bunching by the radiation, strongly influences this coupling. In a low-gain oscillator, the growth of a lower frequency branch results in large induced energy spread, which stops the development of the upper frequency branch. On the other hand, in a high-gain amplifier, a strong signal can be obtained at high frequency by injecting a signal at low frequency that bunches the electron beam without large induced energy spread. In this way, the contradiction about the nonlinear interaction of two resonant waves in a waveguide free-electron laser is explained. [S1063-651X(97)00702-2]

PACS number(s): 41.60.Cr

I. INTRODUCTION

One important feature of the use of a waveguide in a free-electron laser (FEL) is the existence of two different resonant frequencies [1–4] at which the FEL can operate. Then competition and coupling between the upper and lower frequency waves will take place. We have studied the gain [1,2] of a waveguide FEL. Kawamura *et al.* [3] have observed the coupling between two oscillation branches in a waveguide FEL oscillator. They found the wave form of the upper branch is modulated at the rate of the period of the mode-locked oscillation of the lower branch. This phenomenon was explained as the temporally periodic modulation [3,5] of the gain of the upper-branch oscillation by the electromagnetic radiation of the low-branch oscillation through the periodic bunching of the electron beam. They guessed it is impossible for the electron beam to have gain in the upper branch when the electron beam was strongly bunched by the radiation of the lower branch. On the other hand, Piovella *et al.* [4] have theoretically studied the nonlinear space and time interaction between two resonant waves in a high-gain amplifier. They found the unexpected result that a strong signal and bunching at the upper frequency are produced by injecting a signal at the lower frequency in a high-gain FEL amplifier, which means that a bunching of the electron beam by the radiation of the lower frequency results in bunching and gain at the upper frequency. Then there is a contradiction about the nonlinear interaction and coupling between the two resonant waves.

In this paper, we investigate the nonlinear interaction between two resonant waves in a waveguide FEL oscillator. We find that the induced energy spread [6,7] of the electron beam that is produced along with the bunching of the electron beam by the radiation strongly influences the coupling and the development of resonant waves. In a waveguide FEL oscillator, the growth of lower frequency branch results in large induced energy spread of the electron beam, which stops the development of the upper frequency branch because this energy spread is much larger than the maximum allowable energy spread for upper frequency. On the other hand, in a high-gain amplifier, this kind of energy spread even at the saturation of radiation is smaller than the allowable energy spread for the upper resonant waves, so by injecting a signal at lower frequency, which bunches the electron beam without large induced energy spread, a strong bunching and signal can be obtained at the upper frequency particularly when the frequency ratio between them is an integer. In this way, we analyze and explain the contradiction.

II. EQUATIONS

The dispersion relation for the electromagnetic wave in the waveguide is given by $k_s^2=k^2+\Gamma^2$, where k_s and k are the radiation wave number in vacuum and waveguide. For the TE_{01} mode in a plane or rectangular waveguide, $\Gamma=\pi/b$, where b is the waveguide gap. By equating the FEL resonance condition $k_s=\beta(k_w+k)$, where β is the average axial velocity of the electron beam and $\lambda_w=2\pi/k_w$ is the wiggler period, to the dispersion relation, we obtain the expression for the two resonant frequencies [1,3,4].

$$k_{s1,2}=\beta\gamma_z^2k_w(1\pm\beta\Delta) \tag{1}$$

with wave number $k_{1,2}=\beta\gamma_z^2k_w(\beta\pm\Delta)$, where $\gamma_z^2=1/(1-\beta^2),\Delta^2=1-(\Gamma/\beta\gamma_zk_w)^2$.

When the frequency separation between k_{s1} and k_{s2} is larger than the gain bandwidth [1,4], the interference effects between two waves can be ignored. They couple each other through nonlinear interaction with the electron beam. We derived the following set of one-dimensional FEL differential equations [1] to describe this nonlinear interaction:

$$\frac{d\gamma}{dz}=-\frac{a_w}{2\gamma\beta_z}[k_{s1}F_1|a_{s1}|\sin(\theta_1+\varphi_1)+k_{s2}F_2|a_{s2}|\sin(\theta_2+\varphi_2)], \tag{2}$$

$$\frac{d\theta_{1,2}}{dz}=k_w+k_{1,2}-k_{s1,2}/\beta_z, \tag{3}$$

$$\left(\frac{\partial}{\partial z}+\frac{1}{v_{g1,2}}\frac{\partial}{\partial t}\right)a_{s1,2}=i\frac{\omega_b^2 a_w F_{1,2}}{2c^2 k_{1,2}}\langle \exp(-i\theta_{1,2})/\gamma\rangle, \quad (4)$$

$$\beta_z=\sqrt{1-\mu^2/\gamma^2}. \quad (5)$$

In these equations, γ is the electron energy, θ the ponderomotive phase, $a_w=eB_w/(mc^2k_w)$ the dimensionless vector potential of the wiggler, $\mu^2=1+a_w^2/2$, the mass shift, $a_s=|a_s|\exp(i\varphi),|a_{s1,2}|=eE_{1,2}/(mc^2k_{s1,2})$, E is the field amplitude and φ the phase, v_g the group velocity, $F_{1,2}=J_0(\xi_{1,2})-J_1(\xi_{1,2})$, where J is the nth-order Bessel function of the first kind and $\xi_{1,2}=k_{s1,2}a_w^2/(8\gamma^2k_w)$, $\omega_b^2=4\pi e^2n_e/m$ the electron beam plasma frequency squared, $\langle\,\rangle$ represents the average over electrons.

Considering the steady state, i.e., without slippage between the radiation and the electron beam, this limit can be reached with a long enough electron pulse [8] such as that provided by an electrostatic accelerator, Eq. (4) becomes

$$\frac{da_{s1,2}}{dz}=i\frac{\omega_b^2 a_w F_{1,2}}{2c^2 k_{1,2}}\langle \exp(-i\theta_{1,2})/\gamma\rangle. \quad (6)$$

In an oscillator, after one round trip in the cavity resonator, waves will decrease due to the output and the loss in the waveguide wall:

$$a_{s1,2}^{n+1}=q_{1,2}a_{s1,2}^{n}, \quad (7)$$

where $q_{1,2}^2=1-\chi_{1,2}$, $\chi_{1,2}$ is total loss.

III. RESULTS FROM NUMERICAL SIMULATIONS

Equations (2), (3) and (5)–(7) have been integrated using the following parameters of the oscillator experiments [3,5]: $\lambda_w=5$ cm, $B_w=1.44$ kG, number of wiggler periods $N_w=17$, transverse dimensions of the waveguide $a=1.02$ cm and $b=2.29$ cm, electron current $I=2$ A, $\gamma=1.905$ with the maximum small signal gain at $k_{s1}=4.468$ cm^{-1}, $k_{s2}=1.489$ cm^{-1}, and $\alpha=k_{s1}/k_{s2}=3$; $\gamma=1.985$ for $k_{s1}=5.128$ cm^{-1}, $k_{s2}=1.465$ cm^{-1}, and $\alpha=3.5$, $\chi_1=11\%$ and $\chi_2=5\%$ for a coupling hole of 3 mm in diameter and the length of the waveguide of 122 cm, a small power of 0.1 mW for both frequency branches is put in first pass, 256 electrons are used with no initial energy spread.

The numerical simulations confirm the observation of the experiments that the buildup of the lower frequency branch suppresses the development of the upper branch. In Fig. 1, for example, the average power in the cavity at the upper frequency is only about 5% of that at lower frequency, while it should be almost the same as the latter if it developed independently as shown in Fig. 1(c). A less important result is that the evolution of both branches when they couple to each other speeds up because the electron beam is bunched quickly by both waves [4,9]. Furthermore, unlike the case in an amplifier [4], little changes happen as the frequency ratio varies in the range from 3 to 4, even if it is an integer, which again proves the results of the experiments. As an example, we show the results with $\alpha=3$ in Fig. 2; one can see that it is almost the same as Fig. 1.

All preceding results seem to be in contradiction with the theoretical prediction of Piovella *et al.* about a high-gain

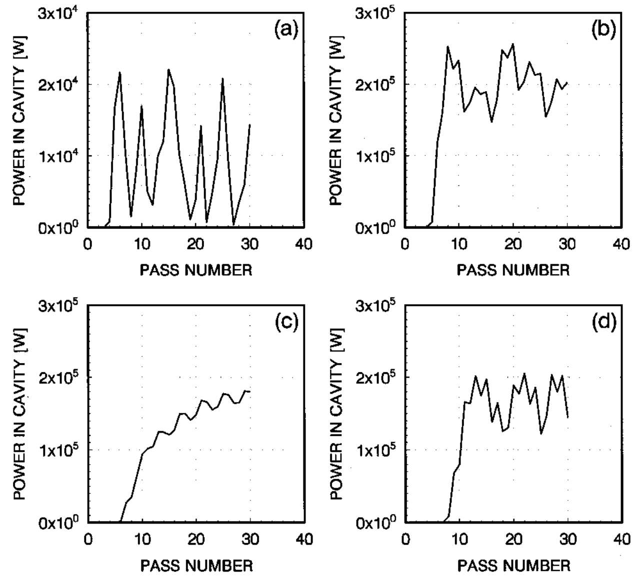

FIG. 1. The development of the power in the cavity at (a) the upper and (b) the lower frequency when they couple with each other; (c) the upper and (d) the lower frequency when they develop independently, respectively, for $\alpha=3.5$.

waveguide FEL amplifier [4]. They found that a strong power and bunching at the upper frequency can be obtained by injecting a signal at a lower frequency when α is an integer.

IV. INDUCED ENERGY SPREAD IN A WAVEGUIDE FEL

The key factor to explain the above paradox is the induced energy spread of the electron beam. The relation between energy loss and induced energy spread is governed by

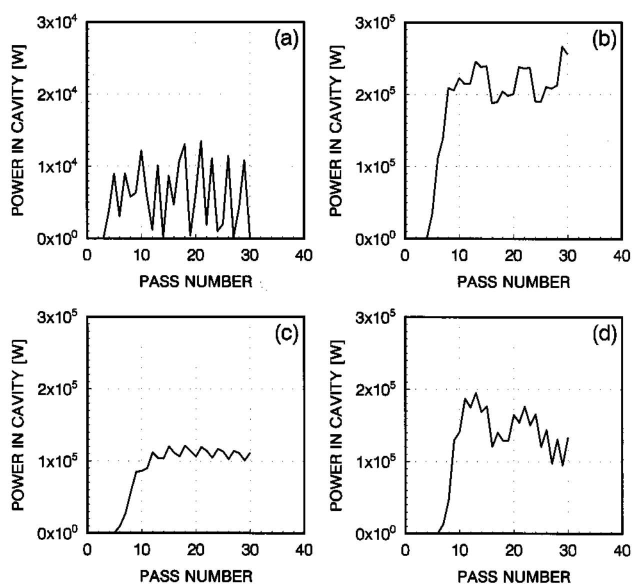

FIG. 2. The development of the power in the cavity at (a) the upper and (b) the lower frequency when they couple with each other; (c) the upper and (d) the lower frequency when they develop independently, respectively, for $\alpha=3.0$.

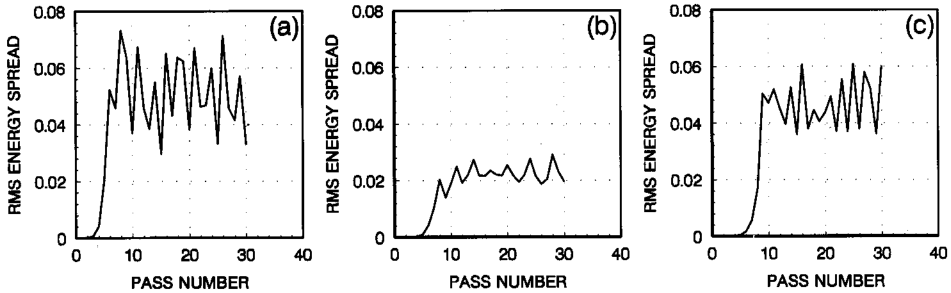

FIG. 3. Final energy spread at each pass, for $\alpha=3.5$ when (a) two waves couple with each other, there is only (b) an upper and (c) a lower frequency wave.

the Madey theorem [6] in the linear regime when the signal is small:

$$\langle \gamma_f - \gamma_i \rangle = \frac{1}{2} \frac{\partial}{\partial \gamma_i} \langle (\gamma_f - \gamma_i)^2 \rangle, \tag{8}$$

where γ_f and γ are final and initial energy. This theorem implies that the induced energy spread is much larger than the mean energy loss as the optical amplitude is small [7], so it increases more quickly than the optical power in former passes. When the saturation is reached in an oscillator with a constant parameter wiggler, the induced energy spread σ_i is on the same order (typically larger) as the mean energy loss σ_l and the maximum allowable energy spread σ_a. They are related by [7]

$$\sigma_i \sim \sigma_l \sim \sigma_a . \tag{9}$$

The maximum allowable energy spread for both resonance frequencies has been given by [2]

$$\sigma_{a1,2} \approx \frac{1}{2N} \frac{2\gamma^2 k_w}{\mu^2 k_{s1,2}}. \tag{10}$$

From relations (9) and (10), if two waves can develop without coupling, we have

$$\sigma_{i1,2} \sim \frac{1}{2N} \frac{2\gamma^2 k_w}{\mu^2 k_{s1,2}} \tag{11}$$

and

$$\sigma_{i2}/\sigma_{i1} = \alpha. \tag{12}$$

When the oscillation of the lower branch can build up in the waveguide cavity resonator with enough gain and a high Q value of the cavity due to the small coupling hole through which it cannot be extracted, the induced energy takes the value of σ_{i2}, which is bigger. This analysis is confirmed by the numerical simulations shown in Fig. 3. Along with the increase of power at lower frequency, the energy spread increases, even faster according to Madey theorem. When it approaches and exceeds the maximum energy spread σ_{a1} for the upper frequency branch, the development of upper branch is suppressed and remains at a quite low level of the power.

From Eqs. (2–6), the Pierce parameter of a waveguide FEL is

$$\rho_{1,2} = \left(\frac{\omega_b^2}{c^2} \frac{a_w^2 \mu^2 F_{1,2}^2}{64\beta^4 \gamma^5 k_w^3} \frac{k_{s1,2}^2}{k_{1,2}} \right)^{1/3}. \tag{13}$$

For highly relativistic electrons, $v_{g1,2} \approx c$, with $F_1 \approx F_2$, one has

$$\rho_1/\rho_2 = \alpha^{1/3}. \tag{14}$$

In a high-gain amplifier, both the efficiency and the induced energy spread will reach the pierce parameter and the maximum allowable energy spread is also about ρ [10]. So the case reverses that in an oscillator; ρ_1 is larger than ρ_2. The energy spread induced by the interaction of the lower frequency wave is much smaller than the allowable energy spread for the upper frequency. A strong bunching on the lower frequency also gives rise to an equally strong bunching on the upper frequency that is integer times of lower frequency, so that a strong signal at the upper frequency can be obtained.

V. DISCUSSION

In conclusion, we have studied the nonlinear interaction between the two resonant waves in a waveguide FEL. We have found that the energy spread induced by radiation strongly influences the development of the two waves. In an oscillator the growth of radiation at the lower frequency produces large induced energy spread and will stop the increase of power at the upper frequency. On the other hand, in an amplifier, the energy spread induced by the lower frequency wave is small because little electron energy is extracted by it. A strong bunching and signal at the upper frequency can be obtained by injecting a signal at the lower frequency.

Recently, Y-H. Liu and T.C. Marshall [11] reported testing of an experiment that tries to obtain millimeter-harmonic generation from a coherent microwave source in a FEL at Columbia University. A quite small signal at the upper frequency was produced along with a gain of 20 at the lower frequency wave. We think the reason is that they operated the FEL at the middle gain regime with a small wiggler field, not the high gain regime; the energy spread of the electron beam and the energy spread is quite large, so the power at the upper frequency is small and the performance of FEL is complicated. This regime needs more careful study.

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